OPTIMAL CONTROL IN HYBRID SYSTEMS

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Abstract: Necessary conditions for optimality have been established for hybrid systems. Unfortunately, these conditions lead to multi-point boundary value problems and they do not prevent from the combinatorial explosion when no constraints are given on the transitions. Different approaches have been proposed to approximate the solution, such as relaxed dynamic programming, non linear programming and sensitivity functions.

Keywords: Hybrid systems, optimal control, sensitivity functions

EXTENDED ABSTRACT

In hybrid systems context, the necessary conditions for optimal control are now well known. These conditions mix discrete and continuous classical necessary conditions on the optimal control. The discrete dynamic involves dynamic programming methods whereas between the a priori unknown discrete values of time, optimization of the continuous dynamic is performed using the maximum principle (MP) or Hamilton Jacobi Bellmann equations(HJB). At the switching instants, a set of boundary tranversality necessary conditions ensure a global optimization of the hybrid system.
These theoretical conditions were applied to minimum time problem and to linear quadratic optimization.
But it is practically very hard to perform such an optimization. The major raison is that discrete dynamic requires evaluating the optimal cost along all branches of the tree of all possible discrete trajectories.
Dynamic programming is then used, but the duration between two switchings and the continuous optimization procedure make the task really hard. This makes the complexity increasing and only problems with a poor coupling between continuous and discrete parts can be reasonably solved.
Nowadays, it seems obvious that only approximated solutions can be found. Various schemes have been imagined.

Recent works have proposed to solve optimal switching problems by using a fixed switching schedule. By switched systems we mean a class of hybrid dynamical systems consisting of a family of continuous (or discrete) time subsystems and a rule (to be determined) that governs the switching between them.
The optimization consists then in determining the optimal switching instants and the optimal continuous control assuming the number of switchings and the order of active subsystems already given.Then a nonlinear search method is used to determine the optimal solution.after the calculus of the derivatives of the value function with respect to the switching instants.
Relaxed Dynamic programming : a relaxed procedure based on upper and lower bounds of the optimal cost was recently introduced. It proved to give good results for piece-wise affine systems and to obtain a suboptimal state feedback solution in the case of a quadratic criteria
Algorithms based on the maximum principle for both multiple controlled and autonomous switchings with fixed schedule have been proposed. The algorithms use the transversality conditions at switching instants. Then, the authors develop a combinational search in order to determine the optimal switching schedule
Interesting results on state or output feedback have been given with the regions of the state space where an optimal mode switch should occur.

In complement of all the methods resulting from the resolution of the necessary conditions of optimality, we propose to use a multiple-phase multiple-shooting formulation which enables the use of standard constraint nonlinear programming methods. This formulation is applied to hybrid systems with autonomous and controlled switchings and seems to be of interest in practice due to the simplicity of implementation.

Sensitivity analysis is the key point of all the methods based on non linear programming. It can also be used to determine limit cycles and the optimal strategy to reach them.

All these items are discussed in the plenary session.

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